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GEOMETRY.

492. Proposed by FRANK V. MORLEY, Student, Haverford College.

Let a_i $(i = 1, 2, 3, 4)$ be four points on a circle, and let the symmedian point of the triangle formed by omitting a_i be s_i. Prove that the four points s_i have the same diagonal triangle as the four points a_i.

493. Proposed by FLORENCE P. LEWIS, Goucher College, Baltimore.

Construct three circles each of which shall be tangent to the other two and to two sides of a given triangle.

494. Proposed by DAVID F. BARROW, University of Texas.

Students of geometry are very apt to assume that a theorem, true in general, will hold in all limiting cases. This trustfulness leads to frequent errors. An example is the following: Let C_1, C_2, C_3, C_4 denote four circles, and P_{ij}, P_{ij}' denote the two points in which C_i and C_j intersect. If P_{12}, P_{23}, P_{34}, P_{41} are concyclic on a circle C, then P_{12}', P_{23}', P_{34}', P_{41}' will be concyclic on a circle C'. This is still true if C is very small. Hence we might hastily conclude that: If four circles are concurrent, then their other intersections, taken in pairs in a cyclic order, are concyclic. Why is not this true?

CALCULUS.

410. Proposed by J. A. BULLARD, Worcester, Massachusetts.

(*a*) Find the area of the loop of the curve $x^{2q+1} + y^{2q+1} = (2q + 1)ax^qy^q$. (For $q = 1$, we have the folium of Calculus Problem No. 379.)

(*b*) Find the area between the curve and its asymptote.

[From Johnson's *Integral Calculus*.]

411. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

Prove that the volume bounded by the surface $f(x, y, z) = 0$ is $\frac{1}{3}\int\int\left(z - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}\right)dxdy$ integrated over the area determined by projecting the surface on the xy-plane.

412. Proposed by CLIFFORD N. MILLS, Brookings, S. Dakota.

Given a triangular field of sides a, b, and c. Show how to divide the field into two equal parts by a straight fence so that the cost of the fence is the least.

MECHANICS.

328. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

Find the envelope of all possible trajectories when a particle is projected with a constant velocity v from a fixed point at a distance a from the center of attraction under the law of gravitation.

329. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A smooth circular table is surrounded by a smooth vertical rim. Show that the ball, whose coefficient of restitution is e, projected along the table from a point in the rim in a direction making an angle $\tan^{-1} e^{\frac{3}{2}}$ with the radius through the point, will return to the point of projection after three rebounds.

NUMBER THEORY.

246. Proposed by ALBERT A. BENNETT, Princeton University.

Prove that

$$\frac{1}{\sqrt{b}}\left[\left(\frac{a+\sqrt{b}}{2}\right)^n - \left(\frac{a-\sqrt{b}}{2}\right)^n\right]$$

is an integer for every positive integral value of n, whenever a is an odd integer, positive or negative, and $b \equiv 1 \pmod 4$.

247. Proposed by NORMAN ANNING, Chilliwack, B. C.

To dissect the triangle whose sides are 52, 56, 60 into three Heronian triangles by lines drawn from the vertices to a point within.

The word Heronian is used in the sense of the German Heronische (Wertheim, *Anfangsgründe d. Zahlenlehre*, p. 140) to describe a triangle whose sides and area are integral.

248. Proposed by E. T. BELL, Seattle, Washington.

If $u_{n+2} = 4u_{n+1} - u_n$, with $u_0 = 2$, $u_1 = 4$, prove that the triangle Δ_n, whose sides are $u_n - 1$, u_n, $u_n + 1$, has an integral area; also that all triangles, Δ_n, whose areas are integers, and whose sides are consecutive integers, are given by this process. Hence show that, as n increases, the area of Δ_n approximates $(\sqrt{3}/4)u_n^2$, and find the degree of approximation.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

447. Proposed by ELIJAH SWIFT, University of Vermont.

(*a*) A method is sought of forming an equation such that the first k figures of some root shall be given numbers. For example, form a cubic equation such that one of its roots is 1.918 +.

(*b*) Of all the equations suggested in (*a*), determine that one for which the sum of the absolute values of all the coefficients is least.

Solution by Paul Capron, U. S. Naval Academy.

Let the degree of the desired equation be n, the nearest integer to the desired root be x_0, and the root itself be $(x_0 + \Delta x)$.

Choose any $\phi(x)$ of degree $(n - 1)$, and form $y = f(x) = (x - x_0)\phi(x)$.

Compute

$$\Delta y = \Delta x \cdot f'(x_0) + \frac{\overline{\Delta x}^2}{\underline{|2}} f''(x_0) + \cdots + \frac{\overline{\Delta x}^n}{\underline{|n}} \cdot f^{(n)}(x_0)$$

from

$$\Delta y = \Delta x \left[\phi(x_0) + \Delta x \cdot f'(x_0) + \cdots + \frac{\overline{\Delta x}^{n-1}}{\underline{|n-1}} \cdot \phi^{(n-1)}(x_0) \right] \tag{1}$$

or from

$$\Delta y = \Delta x \cdot \phi(x_0 + \Delta x). \tag{2}$$

The more convenient of (1) and (2) should be chosen; usually (1) will be preferable. The computation should be carried to at least as many decimals as are required in the root; if it is important that the root should be greater than $(x_0 + \Delta x)$ rather than less, this is readily managed by slightly increasing or decreasing Δy according as $f'(x_0)$ is > 0 or < 0, with due regard to the fact that $\Delta y \doteq \Delta x \cdot \phi(x_0)$. The desired equation is $f(x) - \Delta y = 0$.

If $n = 3$, and $\phi(x) = \alpha_0 x^2 + \alpha_1 x + \alpha_2$,

$$\Delta y = \Delta x[\alpha_0 x_0^2 + \alpha_1 x_0 + \alpha_2 + (2\alpha_0 x_0 + \alpha_1)\Delta x + \alpha_0 \overline{\Delta x}^2].$$

To form a cubic equation having the root 1.918 +, let $\phi(x) = x^2 - 2x + 7$; then

$$f(x) = x^3 - 4x^2 + 11x - 14,$$

and

$$\Delta y = -.082[4 - 4 + 7 + (4 - 2)(-.082) + (1)(.082)^2] = -.5610 -.$$

Increase Δy by .001 to insure a root > 1.918; then $-\Delta y = .560$, and the desired equation is

$$f(x) - \Delta y \equiv x^3 - 4x^2 + 11x - 13.44 = 0,$$

which has a root 1.918 +.

(*b*) If, in the preceding discussion,

$$\phi(x) = \alpha_0 x^{n-1} + \alpha_1 x^{n-2} + \cdots + \alpha_{n-1},$$